3D shape description and matching based on properties of real functions

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Abstract

This tutorial covers a variety of methods for 3D shape matching and retrieval that are characterized by the use of a real-valued function defined on the shape (mapping function) to derive its signature. The methods are discussed following an abstract conceptual framework that distinguishes among the three main components of these class of shape matching methods: shape analysis, *via the application of the mapping function, shape* description, *via the construction of a signature, and* comparison, *via the definition of a distance measure.*
Goal of the tutorial is to facilitate the understanding of the performance of the various methods by a methodical analysis of the properties of various methods at the three different stages.

Categories and Subject Descriptors (according to ACM CCS): I.3.3 [Computer Graphics]: Line and Curve Generation

1. Introduction

3D shape matching and retrieval are key aspects in the current panorama of search engines. Shape models carry a high value with them, and search engines able to retrieve this type of visual media would be surely useful to speed-up content design, re-use and processing. Keyword-based searching is simply not sufficient to achieve the necessary capability of resource exploration for 3D. Therefore, a variety of methods have been proposed in the literature to tackle the problem with different approaches that span from coarse filters suited to browse very large 3D repositories on the web, to domain-specific approaches.

Generally speaking, shape matching methods rely on the computation of a shape *description*, also called signature, that effectively captures some essential features of the object. The shape descriptions are then compared using an appropriate computational technique able to translate the similarity between objects into some distance between descriptors. The majority of the methods proposed in the literature mainly focus on geometric aspects, that is, the description characterizes the spatial distribution or extent of the object in the 3D space [NK01, OFCD02, KFR03]. From a practical point of view, the main advantage of these methods is that they do not make specific assumption on the topology of the digital models and the computational efficiency. Conversely, these methods generally fail in supporting more elaborate shape comparisons, such as partial matching or sub-part correspondence where the similarity has to be evaluated in terms of presence and similarity of features in the shapes. In this case, more sophisticated descriptions should be used, in order to properly characterize the essential features and store them in an efficient and salient structure. Several approaches to shape characterization have been adopted in the literature (e.g. curvature, level-sets, enclosed spheres), yielding to different structuring methods (e.g. patch segmentation, Reeb graph, skeletons, medial axis).

Given the complexity of the problem, understanding and evaluating the performance of methods for 3D matching is not an easy task: first of all, there is neither a single *best* shape characterization nor a single *best* similarity measure, and the solution largely depends on the type of shapes to be analyzed and on the application domains. Recently, a 3D shape retrieval contest has been proposed – SHREC – whose general objective is to evaluate the performances of 3D-shape retrieval algorithms `http://www.aimatshape.net/event/SHREC/`. The initial results of the contest provided the first opportunity to analyze the various algorithms, their strengths, as well as their weaknesses, using a common test collection which allows a direct comparison of algorithms. A single test collection necessarily delivers only a partial view of the whole picture, and for this reason the contest quickly moved towards a multi-track organization, for partial and whole matching, polygon soup and watertight model matching, as well as a number of context-specific benchmarks, for example for mechanical part matching, molecule matching, or 3D face matching.

2. Tutorial focus and contribution

While the performance of retrieval can be evaluated in quantitative terms using appropriate benchmarks and ground truth, it is not easy to understand the contribution to the results of the various components of the retrieval system. The results, indeed, depend both on the shape descriptions and the comparison tools, which are very often quite intertwined. Moreover, existing surveys [BKS*05, TV04, BP06] mainly focus on a classification and discussion of geometry-oriented methods, which target the conversion of statistical and geometric shape analysis into feature vectors or histograms. The comparison among methods usually addresses properties of admissible input representations and formats, invariance of the description with respect to a transformation class, and retrieval performance.

Goal of the tutorial is to facilitate the understanding of the performance of the various methods by a methodical analysis of the properties of various methods at the three different stages of an abstract conceptual framework which distinguishes among the three main components of these class of shape matching methods: shape *analysis*, via the application of some mathematical technique, shape *description*, via the construction of a signature, and *comparison*, via the definition of a distance measure. More precisely, we will analyze in depth methods that approach the analysis phase by making use of the properties provided by some real function f, called the mapping function, defined on the surface $\mathcal{M}$ representing the 3D object. Therefore, the underlying conceptual framework is structured in three-steps:

1. choice and evaluation of the real functions f_i on 3D shapes $\mathcal{M}_i$;
2. construction of *high-level descriptors* $\mathcal{G}_i$ of $\mathcal{M}_i$, using f_i;
3. choice of the comparison techniques to be used for the set of shapes and descriptors $\{(\mathcal{M}_i, \mathcal{G}_i)\}_i$.

We believe that the discussion of the properties at the three levels will facilitate the evaluation of theoretical and practical performances of the methods, will indicate more precisely the strength and weaknesses of the methods, and will also suggest a way for adopting different shape descriptors according to the properties and invariants that one wishes to investigate. The choice of the real function and the nature of the descriptor play indeed the role of the "lens" through which we look at the properties of the shape. The generality and flexibility of the framework is of interest for a wide research community with applications to visualization and topological modeling. In this tutorial, we will overview and analyze a large set of solutions, evaluate their effectiveness, and discuss perspectives, open issues, and future developments.

3. Outline

The proposed tutorial relies on recent survey work of the authors in related fields, see [BFF*06, Mar05, BAB*07].

The updated version of the slides presented at Eurographics 2007 will be made available at the following URL: http://www.ge.imati.cnr.it/ima/smg/training.html

In the following, we outline the main items that we plan to discuss in the tutorial, by giving for each group a synthetic description of the methods and a summary of the most relevant references, which will be discussed in detail and with examples and emphasis on shape matching applications.

3.1. Shape matching: motivations and challenges

The first part of the tutorial will provide an introduction to the tutorial, explain the rationale of the presentation, and introduce some of the main challenges of the topic area and its perspective impact in a number of crucial applications.

3.2. Properties of the real functions

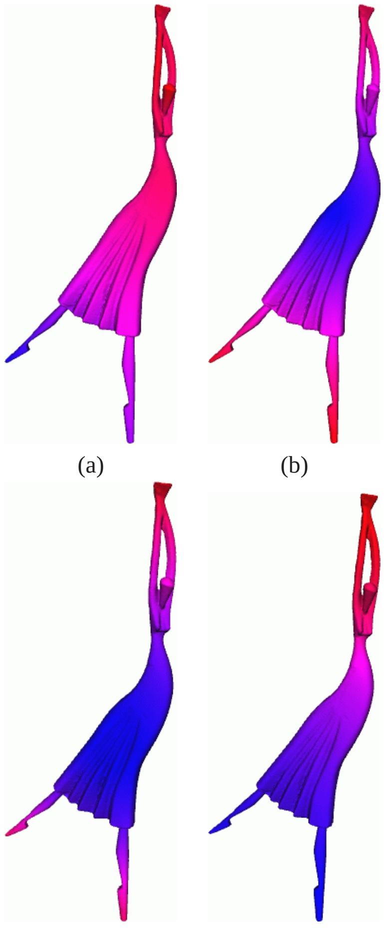
A variety of different functions have been used in the shape matching literature for characterizing relevant features of objects. In general, the availability of *a-priori* information on the classes of the input database can be used to select the mapping functions which are best suited to identify specific shape features (e.g., protrusions), thus constraining the retrieval to match them with a higher degree of importance with respect to other features. This par of the tutorial will provide some introductory definitions on the basic concepts that will be discussed, concerning critical points, Morse function, level sets and briefly introduce their discretization [Ban70, Ban67, GP74, Mil63]. Following, a variety of real-valued functions will be presented and discussed, grouped into four main categories according to their definition, domain and properties:

- the *height* [SKK91, FK97] function is among the most intuitive and simple choices for analysing the shape of an object; since it depends on the direction considered, its usage is preferred for applications in which objects have a natural predefined direction (Figure 1(a)). A more elaborate characterization of the shape according to differences in the elevation value is provided by the *elevation* [AEHW06] function, which derives from the traditional height function but aims at a rotation invariant analysis. The notion of elevation captured by this function measures how much a point is relevant in its normal direction with respect to its neighbourhood. The elevation function is defined by pairing the critical points of the height function in all directions.

- Shape properties can be effectively characterized by measuring distances between feature points or by evaluating the elongation of the shape. In this broad class, the analysis approaches based on the *geodesic distance* generally provide and isometry invariant characterization of a shape [BBK06a]. Geodesic distance has been applied in several settings, in particular for the evaluation the geodesic distance of mesh vertices from selected feature points [MP02, EK03], and for averaging all geodesic distances among the vertices [HSKK01, KT03, GSCO07] . The *Euclidean distance from a point* $\mathbf{p} \in \mathbb{R}^3$ [FK97, SV01] (e.g., the barycentre of $\mathcal{M}$, Figure 1(b)) has also been used, as it is invariant to the shape embedding and detects protrusions (resp. hollows) of $\mathcal{M}$ with respect to $\mathbf{p}$ as regions of influence of maxima (resp. minima) f.
- *curvature-based analysis* have been frequently used to characterize the shape of 3D objects; generally, curvature-based analysis are rather sensible to noise or small features and to the quality of the shape discretization in terms of sampling density and tiny triangles. More robust computation is achieved either using variations of the curvature evaluation function (e.g. [GCO06]), polynomial surface fitting [ZP01], or with a multi-scale curvature evaluation where details are discarded [MPS*04].
- The *local diameters* function [GSCO07] aims at measuring the shape by computing the *diameter* of the volume enclosed by the surface. Therefore, it provides a volumetric rather than a boundary characterization, similarly to the *distance tranforms* [DS06] which is more focused on the medial axis radius.
- If the shapes to be compared do not exhibit a uniform structure, *harmonic* [NGH04, Flo97, PP93] and *Laplacian-based* functions [RWP06, DBG*06] may provide a new and powerful set of descriptors for shape analysis as they are intrinsically defined by the Laplacian matrix of the shape (see Figure 1(c-d)). We will discuss the numerical (in)stability of extraction of this type of functions from the Laplacian matrix of the shape $\mathcal{M}$, a very relevant aspect that has to be considered to understand at which extent this instability affects the descriptor of $\mathcal{M}$, and eventually the matching algorithm [GV89].

The presentation and discussion of the above-cited functions will be carried out considering:

- the *saliency* of f, as its ability to identify relevant shape features of $\mathcal{M}$;
- the *smoothness* degree of f, meant as its behaviour with respect to the number, nature and properties of its critical points;
- the *stability* of f with respect to its discretization and computation on $\mathcal{M}$;



(a) (b) (c) (d)

Figure 1: *(a) Height function, (b) Euclidean distance from the center of mass, (c) harmonic function, (d) first eigenfunction of the Laplacian matrix of the model.*

- the *robustness* of f, that is, the variation of f with respect to small geometric changes of the shape $\mathcal{M}$;
- the *degree of freedom* (DoF) and the number of *heuristics* used in the definition and evaluation of f.
- the *efficiency* of f in terms of the computational cost required by its evaluation on $\mathcal{M}$;
- the *invariance* of f to transformation groups;
- the hypotheses or restrictions on the *input*.

critical points of $(\mathcal{M}, f)$ and measure the smoothness of a function f through its *Sobolev norm* [GV89, CM06] and other differential descriptors. The analysis of the properties and the potentialities of the fs will provide an insight into the formalization of function suites, beyond a generic best-practice or rule-of-thumbs.

3.3. Properties of the shape descriptors

In the literature, it is quite common that functions used to analyse the shape are directly associated to a corresponding signature, or shape descriptor. For some of the methods this association is exclusive, meaning that no other function can be used to produce the same descriptor, while for other methods the descriptor is *parametric* with respect to the choice of the function.

Among shape descriptors that are parametric with respect to the choice of f, we will present:

- Reeb graphs [Ree46, CMEH*03, HSKK01, ABS03, Bia04, TS05, BFS00], size theory [Fro90, FL01, FL99, dFL06, FM99, BCF*07] and persistent homology tools [ELZ02, CZCG04, CZCG05, WAB*05, ZC05, CSEH05, CSEH07] are topological descriptors that root in Morse theory. When the function f varies, a collection of descriptors may be obtained. For any f, these descriptors code the shape by the configuration of elements or properties that characterize the topological evolution of level sets or lower level sets of f, see Figure 2;
- descriptors that decompose a function f given over simpler basis functions; examples are the spherical harmonic shape decompositions [KFR03, Vra04, VSR01] and wavelets-based methods [LTN06].

Among shape descriptors that exclusively linked to a specific choice of f, we will present:

- descriptors based on quantities extracted by intrinsic shape functions, such as the spectrum of the Laplace-Beltrami operator [RWP06, RWP07, NRW*07];
- descriptors built on isometry invariant quantities, as for example the geodesic function [JZ06, JZ05, EK03, BBK05, BBK06b, BBK06a] or the curvature [ZP01, GCO06];
- the pose-oblivious shape signature [GSCO07], that associate to $\mathcal{M}$ histograms of the distribution over the shape of two real functions, the first related to surface and the second to volume information;
- the centerline skeleton that connects feature points through the geodesic distance [MP02])

The shape descriptors will be presented from a theoretical and computational point of view, providing examples and results to assess different aspects, in particular:

- the *saliency* of the descriptor, that is its ability to capture the structure of the shape in terms of its features;
- the *concisness* of the descriptor, that is its ability to minimize the memory needed to store the descriptor while maximizing the amount of information represented; this property is related also to the type of output produced;
- the *robustness* with respect to small changes of the shape;
- the *unicity* of the descriptor: once the theoretical methodology for extracting the descriptor, the algorithm, and possible parameters have been chosen, the descriptor is unique;

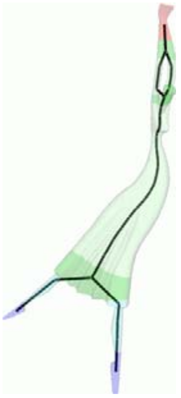


Figure 2: *(a) Reeb graph of the first eigenfunction of the Laplacian matrix of the model and (b) of the Euclidean distance from the center of mass.*

- the *completness* in the sense that the same descriptor cannot be associated to different shapes;
- the *invariance* of the descriptor to transformation groups;
- the *degree of freedom* (DoF) and the number of *heuristics* used in the construction of the descriptor;
- the hypotheses or restrictions on the *input*;
- the *efficiency* of the descriptor in terms of the computational cost required by its construction.

3.4. Comparison methodologies

Although the surveyed descriptors are inspired by the same idea of quantifying geometric properties conveyed by f, there are substantial differences in the shape interpretation they provide and in the structures used to encode the shape information. In particular, the type of structure produced strongly influences the choice of the methods adopted for the final shape comparison step. The methodologies will be presented following a logical grouping according to the type of coding of the shape descriptor:

- the similarity between descriptors encoded as *histograms, feature vectors, or matrix structures* is evaluated by linear algebraic or statistical techniques [KFR03, Vra04, LTN06];
- the similarity among descriptors stored as *graphs* is generally evaluated by graph-matching techniques [HSKK01, SSGD03, LK03, CDS*05, BSRS04, ZSm*05, BRS06, BMSF06] (see Figure 3).
- the similarity between combinatorial descriptors is measured by friendly and computationally efficient tools, such as persistence diagrams and formal series [dFL06, BCF*07, CSEH07].

The methodologies will be presented and discussed highlighting their properties in terms of the following characteristics:

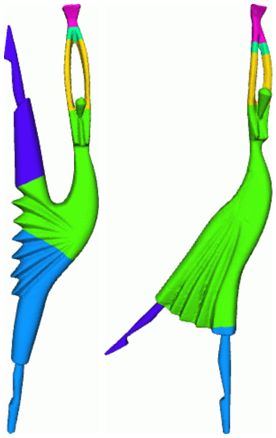


Figure 3: *Sub-part correspondence obtained using the graph comparison method defined in [BMSF06].*

- the *properties* of the similarity measure that characterize it as a metric, semi-metric, or pseudo-distance [VH01, Tve77, SJ99];
- the *robustness* of the measure with respect to small changes of the shape;
- the *type of comparison* provide by the measure, in terms of supporting global, partial or sub.part correspondence;
- the *type of information*: according to the type of information stored and the way it is coded in the descriptor, the measure of similarity may take into account geometric, topological or structural information;
- the *efficiency* in terms of computational complexity required to evaluate the measure;
- the *application scenario* in which the comparison is performed.

3.5. Conclusions and future perspectives

In the conclusive part of the tutorial, we will try to provide a coherent comparison of the various techniques at the three levels of the framework, based on the analysis provided for all the aspects discussed. Obviously, the tutorial does not claim either to be an exhaustive survey of the wealth of existing methods for 3D matching or to examine all technical details of each single method. Rather, the objective of the comparison is to give a structured presentation of the methods in terms of the several properties of the descriptors and comparison tools, that are often not discussed in details in existing surveys. We believe that the presentation and discussions organized in this manner should serve as a basis for extending the performance analysis beyond standard precision-recall diagrams and help the user to understand if the reasons of good or bad retrieval results depend, for instance, on an insufficient efficacy of the descriptor, on an intrinsic instability of the function, or also on an inappropriate comparison tool.

Finally, we will list a series of topics deserving further research, such as the role of invariance with respect to transformation groups, the concurrent use of more than a single characterizing function, and the need to balance the use of geometrical and topological information for accurate shape descriptions. Last but not least, we will also address issues related to the emerging use of semantic indicators to perfom matching and retrieval, based either on (semi)-automatic annotation of shapes or in supervised classification and prototype extraction.

4. Authors' CVs

Two research groups are involved:

The *Shape Modeling Group at CNR-IMATI-GE* works since years on topics related to geometric modelling with the main aim to describe the shape of objects through geometric and topological reasoning techniques. Lately, the research themes focus on broadening the role of traditional modelling with the definition of new representations, encapsulating also knowledge technologies methodologies, able to express also the semantic level at which the perception of shape is encoded. In this field, CNR-IMATI-GE is leading the FP6 European Project NoE AIM@SHAPE.

The team *Vision Mathematics of the Univ. of Bologna*, Dept. of Mathematics, works at the use of topology and geometry in robotic applications since 1988. Mainly, the team deals with computer vision by means of a shape descriptor (the Size Functions) conceived and developed by P. Frosini. But the group interests cover a fairly wide area reaching from the abstractions of manifold topology to robot navigation and to concrete application projects.

Bianca Falcidieno is Research Director at CNR and head of the Shape Modelling Group, working in the field of Applied Mathematics and Computer Science, with applications in Computer Graphics, Geographic Information Systems, and Industrial Design. She is Editor in chief of the International Journal Shape Modelling, member of the Steering Committee of Shape Modeling International (SMI), and author of more than 200 scientific refereed papers and books. Bianca Falcidieno is the coordinator of the FP6 NoE AIM@SHAPE.

Patrizio Frosini is assistant professor in the Faculty of Engineering at the Univ. of Bologna. He is a member of the ARCES group at the Univ. of Bologna. He received the PhD degree in Mathematics from the Univ. of Florence (1991). His research interests include the study of geometrical-topological methods for shape comparison and related applications in Computer Vision.

Claudia Landi is assistant professor at the Univ. of Modena and Reggio Emilia in Reggio Emilia (Italy). She obtained a PhD in Mathematics in 2000, at the University of Pisa. Since 1994 she is member of the Vision Mathematics Group of the University of Bologna. Her main research interest is shape description via geometry and topology.

Michela Spagnuolo is senior researcher at CNR-IMATI-GE and received the Ph.D. in Computer Science Engineering, at the INSA, Lyon, France (1997). Her research interests are related to shape-based approaches to modeling digital shapes, computational topology techniques for shape analysis, geometric reasoning for the extraction of shape features from discrete surface models, and geometric models for coding uncertainty in data samples (fuzzy-based modelling). She is a member of the Steering Committee of Shape Modeling International (SMI).

Silvia Biasotti is researcher at CNR-IMATI-GE and received a Ph.D. in Mathematics and Applications at the Univ. of Genoa (2004). Her research interests include computational topology, shape abstraction and skeleton representation of polyhedral surfaces.

Daniela Giorgi is research fellow at CNR-IMATI-GE and received a Ph.D. in Applied Mathematics at the Univ. of Padua (2006). Her research interests are in Pattern Recognition and topological methods for shape analysis.

Simone Marini is researcher at CNR-IMATI-GE and received a Ph.D. in Electronic and Computer Engineering at the Univ. of Genova (2005). His main interests concern evaluation of 3D shape similarity, graph comparison, and ontological representation of scientific concepts.

Giuseppe Patané is researcher at CNR-IMATI-GE and received a Ph.D. in Mathematics and Applications at the Univ. of Genova (2005). His research interests include numerical analysis (implicit surfaces), shape analysis, computational geometry (topological graphs, local and global parameterization).

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